Influence of ion temperature on Lyman-α intensity ratios

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In this paper we examine the effects of varying ion temperature with constant electron temperature on the Lyman-α intensity ratios for a small admixture of hydrogenlike ions in a high-temperature plasma. We investigate the ratios with increasing central electron temperature, change in base plasma, and increasing nuclear charge, Z, of the impurity ion. [S1063-651X(99)01805-X]

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I. INTRODUCTION

The ion temperature in a plasma can often be considerably higher than the electron temperature, especially in tokamaks. However, the influence of the ion temperature is not seen until higher densities than those usually observed in tokamaks ($n_e \sim 10^{13}-10^{14}$ cm^{-3}). The ion collisions are mainly those inducing transitions between the $n-n'$ states, where both n and $n'>1$, i.e., collisions to and from the ground state, are not induced by heavy ion impact. Also of relevance are collisions which produce the transitions $(n,l,j)-(n,l',j')$, i.e., $\Delta n=0$, where $n>1$. Since we consider the Lyman-α intensity ratios, it is the $2P$ sublevels which are relevant, their populations being significantly affected by ion collisions.

The Lyman-α intensity ratio is the ratio of the intensities of the transitions $2P_{1/2}\rightarrow 1S_{1/2}$ and $2P_{3/2}\rightarrow 1S_{1/2}$. In this paper, we examine the effect of varying the ion temperature around a constant electron temperature on the intensity ratio, under a variety of different plasma conditions. Previous theoretical analyses of the intensity ratios have assumed equal ion and electron temperatures [1,2] since the published results used from [3,4] use a program with this assumption maintained. We have modified this program [5] to distinguish between the ion and electron temperatures, and the results presented in this paper appear to be the first ones mentioned in the literature.

II. VARIATION OF THE ION TEMPERATURE IN A HYDROGEN PLASMA

The program used to calculate the level populations and hence the intensity ratios, COLRAD [5], uses a collisional-radiative model to determine the excited level populations for hydrogenlike ions present as a small admixture in a high-temperature plasma. We include the contribution from the magnetic dipole transition, $2S_{1/2}\rightarrow 1S_{1/2}$, in our values [6] for the Lyman-α intensity ratios as this cannot be resolved experimentally from the $2P_{1/2}\rightarrow 1S_{1/2}$ transition, and we present our results here for comparison with both past and future experimentally measured intensity ratios.

In this section we consider hydrogenlike Al XIII present in a hydrogen base plasma, although we also examine the effect on the intensity ratios of changing the base plasma to one of higher Z in Sec. V. Therefore, for Al XIII, we considered three cases for the constant electron temperature, with $T_e=1$, 4, and 10 keV. The ion temperature T_i was varied in magnitude around each constant electron temperature, T_e, by factors of 2, 3, and 4, and then by factors of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. The bold line in Fig. 1 is the curve for the intensity ratios against electron density, n_e, with equal ion and electron temperatures.

Figure 1 shows that as T_i is increased above T_e, there is a significant decrease in the intensity ratios, which becomes successively larger in magnitude as the difference between the ion and electron temperatures increases. Conversely, when T_i is reduced below T_e, the intensity ratios increase above their values at equal ion and electron temperatures, again with a successively larger increase in magnitude as the difference between the ion and electron temperatures increases. These effects are explained by the fact that over this temperature range, the rates for the proton impact transition $2S_{1/2}-2P_{1/2}$ decrease with temperature. This influence on the intensity ratios is most pronounced for the electron density range $n_e=10^{15}-10^{20}$ cm^{-3}. Outside this density range, the effect is minimal, of the order of 0.3% or so.

Table I shows the percentage differences in the intensity ratios from the values when $T_i=T_e$ for the pure hydrogen base plasma case, as considered in this section, with varying ion temperature. The reduction in the ratios becomes larger

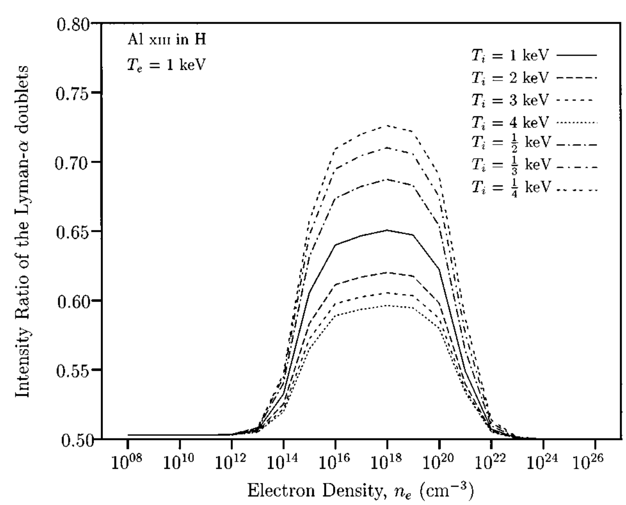

FIG. 1. A graph of the intensity ratios against n_e for Al XIII in a hydrogen base plasma, with T_i varied around constant $T_e=1$ keV, as shown.

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TABLE I. Table of percentage differences (from the values when $T_i=T_e$) in the intensity ratios for Al XIII against n_e with T_i varied around constant T_e, as below, for a hydrogen base plasma.

Factor \ n_e (cm^{-3})	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
			$T_e=1$ keV			
2	−4%	−4%	−5%	−5%	−5%	−4%
3	−6%	−7%	−7%	−7%	−7%	−6%
4	−7%	−8%	−8%	−8%	−8%	−7%
$\frac{1}{2}$	4%	5%	6%	6%	6%	5%
$\frac{1}{3}$	7%	9%	9%	9%	9%	8%
$\frac{1}{4}$	8%	11%	11%	12%	12%	11%
			$T_e=4$ keV			
2	−2%	−3%	−3%	−3%	−3%	−2%
3	−4%	−4%	−4%	−5%	−4%	−4%
4	−4%	−5%	−5%	−5%	−5%	−4%
$\frac{1}{2}$	3%	4%	4%	4%	4%	3%
$\frac{1}{3}$	5%	6%	6%	7%	6%	5%
$\frac{1}{4}$	7%	8%	8%	9%	8%	7%
			$T_e=10$ keV			
2	−2%	−2%	−2%	−2%	−2%	−2%
3	−2%	−3%	−3%	−3%	−3%	−2%
4	−3%	−3%	−3%	−3%	−3%	−2%
$\frac{1}{2}$	2%	3%	3%	3%	3%	2%
$\frac{1}{3}$	4%	4%	5%	5%	5%	4%
$\frac{1}{4}$	5%	6%	6%	6%	6%	5%

as the multiplication factor for the ion temperature, T_i/T_e (i.e., the factor by which the electron temperature is multiplied to obtain the ion temperature), increases from 2 to 4. A maximum reduction of the ratios by 8% for $n_e=10^{16}–10^{19}\,cm^{-3}$ is produced when the ion temperature is four times the electron temperature, with $T_e=1$ keV. There is a similar increase in the ratios when the ion temperature is less than the electron temperature, and the magnitude of this enhancement increases as the multiplication factor is decreased. This time a maximum increase of the ratios of 12% over the density range $10^{18}–10^{19}\,cm^{-3}$ is obtained when $T_i=\frac{1}{4}T_e$ with $T_e=1$ keV.

Hence, we note that the maximum effect on the intensity ratios of varying the ion temperature is observed at lower electron temperatures, and gradually has less influence as the electron temperature increases. This is as a result of the plasma approaching closer to local thermodynamic equilibrium (LTE) at higher temperature.

III. VARIATION OF THE BASE PLASMA

When the base plasma was changed from pure hydrogen to pure oxygen, with $T_e=1$ keV and the same impurity ion, Al XIII, Table II shows that the effect of varying the ion

TABLE II. Table of percentage differences (from the values when $T_i=T_e$) in the intensity ratios for Al XIII against n_e with T_i varied around constant T_e, as below, for an oxygen base plasma.

Factor \ n_e (cm^{-3})	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
			$T_e=1$ keV			
2	1%	−0.3%	−0.4%	−1%	−1%	−2%
3	1%	−1%	−1%	−1%	−2%	−3%
4	0.3%	−1%	−2%	−2%	−3%	−5%
$\frac{1}{2}$	−3%	−1%	−1%	−0.3%	0.1%	1%
$\frac{1}{3}$	−5%	−2%	−1%	−1%	−0.5%	0.3%
$\frac{1}{4}$	−8%	−3%	−3%	−2%	−1%	−0.4%
			$T_e=4$ keV			
2	−2%	−2%	−2%	−3%	−3%	−4%
3	−4%	−4%	−4%	−5%	−5%	−7%
4	−5%	−6%	−6%	−6%	−7%	−9%
$\frac{1}{2}$	1%	1%	1%	2%	2%	3%
$\frac{1}{3}$	0.3%	2%	2%	2%	3%	5%
$\frac{1}{4}$	−0.2%	2%	2%	3%	3%	6%

TABLE III. Table of percentage differences (from the values when $T_i = T_e$) in the intensity ratios for Al XIII against n_e with T_i varied around constant T_e, as below, for an aluminum self-plasma.

Factor \ n_e (cm^{-3})	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
			$T_e = 1$ keV			
2	2%	1%	0.4%	0.1%	−0.2%	−1%
3	3%	1%	0.3%	−0.2%	−1%	−2%
4	3%	0.3%	0.01%	−1%	−1%	−3%
$\frac{1}{2}$	−5%	−2%	−2%	−1%	−1%	−0.5%
$\frac{1}{3}$	−8%	−4%	−3%	−3%	−2%	−2%
$\frac{1}{4}$	−12%	−6%	−5%	−5%	−4%	−3%
			$T_e = 4$ keV			
2	−1%	−1%	−1%	−2%	−2%	−3%
3	−1%	−2%	−3%	−3%	−3%	−6%
4	−2%	−3%	−4%	−4%	−5%	−8%
$\frac{1}{2}$	−1%	0.4%	1%	1%	1%	2%
$\frac{1}{3}$	−2%	0.3%	1%	1%	2%	3%
$\frac{1}{4}$	−3%	0.1%	0.4%	1%	2%	4%

temperature as before is noticeably smaller on the intensity ratios, as well as a slight ''crossover'' effect, where for an integer multiplication factor there is a slight enhancement of the ratios at the lower end of the affected density range before the expected reduction in the ratios occurs. Correspondingly, for a fractional multiplication factor, there is a slight reduction at the upper end of the density range, prior to which the expected enhancement in the ratios has been observed. However, when the electron temperature is increased to 4 keV, this ''crossover'' effect is no longer observed and the respective enhancements and reductions in the intensity ratios are correlated with the multiplication factor being a fraction or an integer, respectively.

The final analysis on differing the base plasma was for a pure aluminum self-plasma, such as that produced by a laser. We considered the cases when $T_e = 1$ and 4 keV as before. When $T_e = 1$ keV, the effect on the intensity ratios is again smaller than the case when the base plasma is pure hydrogen (see Table III). Over the significant density range, 10^{16}–10^{19} cm^{-3}, inverse behavior is observed to that for pure hydrogen. The fractional multiplication factors produce a reduction which is a maximum for $T_i = \frac{1}{4} T_e$ with 12% for $n_e = 10^{15}$ cm^{-3}. The integer multiplication factors, however, again produce the same ''crossover'' effect that was observed in the pure oxygen plasma case. The intensity ratios are initially enhanced at $n_e = 10^{15}$ cm^{-3}, but as the density increases, this becomes a reduction in the values which peaks at $n_e = 10^{21}$ cm^{-3}, with a maximum value of 7% for a multiplication factor of 4.

When the electron temperature is 4 keV, the effect of varying the ion temperature by the given multiplication factors reverts back to being the same as occurred for the hydrogen base plasma, except at $n_e = 10^{15}$ cm^{-3}, when for the fractional multiplication factors the ''crossover'' effect is

TABLE IV. Table of percentage differences (from the values when $T_i = T_e$) in the intensity ratios for Cl XVII and Cr XXIV against n_e with T_i varied around constant T_e of 2 keV for Cl XVII and 4 keV for Cr XXIV, respectively, as below, for a hydrogen base plasma.

Factor \ n_e (cm^{-3})	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
			Cl XVII			
2	−1%	−4%	−5%	−5%	−5%	−5%
3	−2%	−5%	−7%	−7%	−7%	−7%
4	−2%	−7%	−8%	−9%	−9%	−9%
$\frac{1}{2}$	1%	4%	5%	6%	6%	6%
$\frac{1}{3}$	2%	6%	9%	9%	9%	9%
$\frac{1}{4}$	2%	8%	11%	11%	12%	11%
			Cr XXIV			
2	−0.1%	−0.5%	−3%	−5%	−5%	−5%
3	−0.1%	−1%	−4%	−7%	−8%	−8%
4	−0.1%	−1%	−5%	−9%	−9%	−10%
$\frac{1}{2}$	0.04%	0.5%	3%	5%	6%	6%
$\frac{1}{3}$	0.1%	1%	4%	8%	9%	9%
$\frac{1}{4}$	0.1%	1%	5%	9%	10%	11%

still observed, and the intensity ratios are mildly reduced. For all other densities in the given range, the expected enhancement and reduction for the fractional and integer multiplication factors, respectively, are seen.

IV. VARIATION OF THE IMPURITY ION

The final analysis was a systematic calculation of the intensity ratios for hydrogenlike ions of increasing Z under the same conditions with the reduced electron temperature (T_e/Z^2) kept constant and T_i varied as before. The three ions considered were the original test impurity ion (Al XIII), Cl XVII, and Cr XXIV; T_e was 1 keV for Al XIII, 2 keV for Cl XVII, and 4 keV for Cr XXIV.

Tables I and IV show that the effect of varying the ion temperature by a fractional multiplication factor decreases with increasing Z, but that this decrease is most significant for the case when $T_i = \frac{1}{4} T_e$. As for the pure oxygen base plasma, the percentage differences in the intensity ratios for Cl XVII and for Cr XXIV are still increasing at the upper end of the density range considered.

V. CONCLUSION

Our systematic analysis shows that varying the ion temperature from the electron temperature can produce a significant effect on the intensity ratios. The extent of this effect varies according to other plasma conditions, most notably the actual electron temperature itself, the type of base plasma, and the nuclear charge of the ion considered. When the base plasma is changed from pure hydrogen to pure oxygen, the absolute magnitude of the difference between the intensity ratios for equal ion and electron temperatures and those for the specified varied ion temperature is considerably smaller.

In a hydrogen base plasma, the majority of the heavy ion collisions involve protons which play a significant role in the collisions producing population transfer between the $2P$ sublevels. From the calculations which determine the rate coefficients for transitions between n,l,j sublevels within the same principal quantum shell due to heavy ion impact, it is found that increasing T_i above T_e produces a significant change in these rate coefficients. In particular, those of direct relevance to the populations of the $2P_{1/2}$ and $2P_{3/2}$ levels are the rate coefficients for the population transfer from the $2S_{1/2}$ level to these two levels. Increasing T_i above T_e causes the rate coefficient for the $2S_{1/2} \rightarrow 2P_{1/2}$ transition to be decreased while the rate coefficient for the $2S_{1/2} \rightarrow 2P_{3/2}$ transition is increased by a greater amount in magnitude. Thus it can be seen that the subsequent overall decrease in the intensity ratios with increasing ion temperature at constant electron temperature can be explained (see Fig. 1). Similarly, calculations of the rate coefficients when T_i is reduced below T_e show that the rate coefficient for the $2S_{1/2} \rightarrow 2P_{1/2}$ transition is increased, while the rate coefficient for the $2S_{1/2} \rightarrow 2P_{3/2}$ transition is reduced by a greater amount in magnitude. This therefore produces the observed overall increase in the intensity ratios with decreasing ion temperature at constant electron temperature.

The rate coefficients for transitions induced by heavy ion collisions are inversely proportional to $T_i^{1.5}$ [5], and hence it can be seen that increasing T_i above T_e, e.g., when $T_e = 1$ keV, will reduce the rate coefficients and reducing T_i below T_e will enhance them. The effects observed with the change of base plasma from hydrogen to oxygen, for example, are due to the fact that the rate coefficients are also dependent on the heavy ion mass in a complex function [5], and together with effects caused by varying T_i, these cause the intensity ratios to vary as shown in Table II. When the base plasma is oxygen, calculations of the rate coefficients for the transitions between the $2S_{1/2}$ and $2P$ sublevels show that the rate coefficients for both the $2S_{1/2} \rightarrow 2P_{1/2}$ transition and the $2S_{1/2} \rightarrow 2P_{3/2}$ transition are now increased when T_i is increased above T_e, and reduced when T_i is decreased below T_e. However, in each case, it is the rate coefficient for the $2S_{1/2} \rightarrow 2P_{3/2}$ transition which has either a significantly larger increase or reduction compared to that for the $2S_{1/2} \rightarrow 2P_{1/2}$ transition. There is an additional dependence of the rate coefficients on the nuclear charge of the impurity ion considered which is again rather complex [5], and this gives rise to the observed differences in the enhancements and reductions in the intensity ratios (see Table IV) as Z increases from Al XIII through Cr XXIV. A fuller explanation of the "crossover" effect observed in the results for the high-Z base plasmas requires further analysis of the dependence of the relevant cross sections on Z.

Thus our implementation in COLRAD of distinct electron and ion temperatures in a high-temperature plasma might go some way towards explaining previous differences noted [1,2] between experimental and theoretical intensity ratios, as the experimental ion temperature is indeed frequently different from the experimental electron temperature in both tokamaks and laser-produced plasmas.

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